Mr. Michael T. Davis Calculus	Section 3.4 Practice Quiz January 9, 2017

Name: Solutions

1. Consider the volume of a sphere.

   a. Write the volume of a sphere as a function of its radius r.

   $$V(r) = \frac{4}{3}\pi r^3$$

   b. Find the instantaneous rate of change of the volume V with respect to radius r of the sphere.

   $$\frac{dV}{dr} = 3 \cdot \frac{4}{3}\pi r^2 = 4\pi r^2$$

   c. Evaluate the rate of change of the volume V at $r=1$.

   $$\frac{dV}{dr}(1) = 4\pi(1)^2 = 4\pi$$

   d. If r is measured in inches and V is measured in cubic inches, what units would be appropriate for $\frac{dV}{dr}$

   $$\frac{dV}{dr} = \frac{in^3}{in}$$

2. The coordinates of a moving body for various values of t are given in the table.

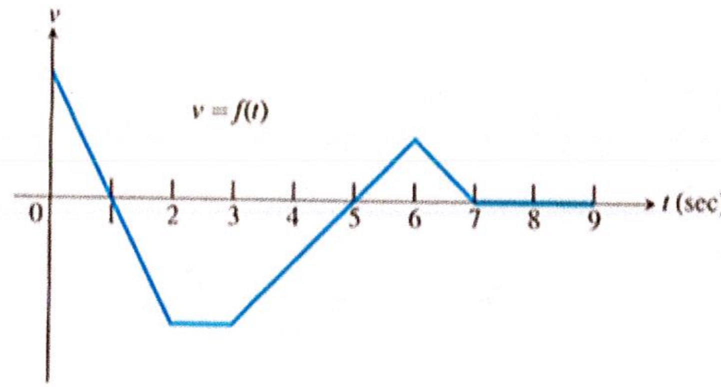
t (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s (ft)	12.5	26	36.5	44	48.5	50	48,5	44	36.5

   a. Plot s versus t on coordinate paper, and sketch a smooth curve through the given points.

   b. Assuming that this smooth curve represents the motion of the body, estimate the velocity of the moving body at $t=1.5$. Explain your work.

   $$V(1.5) \approx \frac{s(2)-s(1)}{2-1} = \frac{48.5-36.5}{1} = 12 \ \text{ft/sec}$$

3. Particle Motion. The accompanying figure shows the velocity $V = f(t)$ of a particle moving on a coordinate line.



a. When does the particle move forward? Justify your answer.

$0 \le t < 1$, $5 < t < 7$ since $V(t) > 0$

b. When does the particle move backward? Justify your answer.

$1 < t < 5$ since $V(t) < 0$

c. When does the particle speed up? Justify your answer.

$1 < t < 2$, $5 < t < 6$ $|v(t)|$ is increasing which means speed is increasing

d. When does the particle slow down? Justify your answer.

$0 < t < 1$, $3 < t < 5$, $6 < t < 7$ since $|v(t)|$ is decreasing

e. When is the particle's acceleration positive? Justify your answer.

acceleration is the derivative of velocity

$3 < t < 6$ since $a(t) > 0$

f. When is the particle's acceleration negative? Justify your answer.

$0 < t < 2$, $6 < t < 7$ since $a(t) < 0$

g. When is the particle's acceleration zero? Justify your answer.

$2 < t < 3$, $7 < t < 9$ since $a(t) = 0$

h. When does the particle move at its fastest speed? Justify your answer.

either at $t = 0$ or $2 < t < 3$ since $|v(t)|$ is the greatest at these times

i. When does the particle stand still for more than an instant? Justify your answer.

$7 < t < 9$ since $V(t) = 0$

4. Lunar Projectile Motion. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s(t) = 24t - 0.8t^2$ meters in t seconds.

a. Find the rock's velocity as a function of time.

$v(t) = 24 - 1.6t$ m/sec

b. Find the rock's acceleration as a function of time.

$a(t) = -1.6$ m/sec^2

c. How long did it take the rock to reach its highest point? Justify your answer.

$v(t) = 0 \quad 0 = 24 - 1.6t \quad t = \frac{24}{1.6} = 15$ sec

$v(t)$: $-\,-\,-\,-$ (0 to 15), $+\,+\,+\,+$ (15 to 30); marks 0, 15, 30

$v(15) = 0$ and $v(t)$ changes from negative to positive at $t = 15$

d. How high did the rock go?

$s(15) = 24(15) - 0.8(15)^2 = 180$ m

e. When did the rock reach half its maximum height? Justify your answer.

$s(t) = 90$ m $\quad 90 = 24t - 0.8t^2$

at $t \approx 4.39$ sec and $t \approx 25.61$ sec

f. For how long was the rock aloft? Justify your answer.

For 30 sec since the rock takes 15 sec to reach its max height and 15 sec to return to the ground, and $s(30) = 0$ m

5. Particle Motion. The position of a body at time t sec is $s(t) = t^3 - 3t^2 + 3t$ m. Find the body's acceleration each time the velocity is zero.

$v(t) = 3t^2 - 6t + 3 = 3(t^2 - 2t + 1) = 3(t-1)^2$

$0 = 3(t-1)^2 \quad t = 1$ sec

$a(t) = 6t - 6 \quad a(1) = 0$ m/sec^2

6. Particle Motion. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^3 - 6t^2 + 8t + 2$ where s is measured in meters and t is measured in seconds.

   a. Find the instantaneous velocity at any time t.

   $v(t) = 3t^2 - 12t + 8$ m/sec

   b. Find the acceleration of the particle at any time t.

   $a(t) = 6t - 12$ m/sec^2

   c. When is the particle at rest? Justify your answer.

   $v(t) = 0 \quad 0 = 3t^2 - 12t + 8 \qquad t = \frac{12 \pm \sqrt{144 - 4(3)(8)}}{2(3)}$

   $t \approx 0.845$ and $t \approx 3.155$

   d. Describe the motion of the particle. At what values of t does the particle change directions? Justify your answer and explain why.

   the velocity changes from Pos to Neg at $t \approx 0.845$ and the velocity changes from Neg to pos at $t \approx 3.155$

   $v(t)$: + + (0 to 0.845), − − − (0.845 to 3.155), + + + (after 3.155)

   The particle moves to the right at $t = 0$, changes direction to the left at $t \approx 0.845$ sec then changes direction again at $t \approx 3.155$ sec and continues moving to the right.

7. Marginal Cost. Suppose the dollar cost of producing x cellular phones is $c(x) = 1500 + 80x - 0.2x^2$.

   a. Find the average cost of producing 75 cellular phones.

   $\frac{c(75)}{75} = \frac{1500 + 80(75) - 0.2(75)^2}{75} \approx \$85$ per phone

   b. Find the marginal cost when 75 cellular phones are produced.

   Marginal Cost is the derivative of the cost function.

   $c'(x) = 80 - 0.4x \qquad c'(75) = 80 - 0.4(75) = \$50$ per the next phone / the 76th phone

   c. Show that the marginal cost when 75 cellular phones are produced is approximately the cost of producing one more cellular phone after the first 75 have been made, by calculating the latter directly.

   $c(76) - c(75) = 1500 + 80(76) - 0.2(76)^2 - \left(1500 + 80(75) - 0.2(75)^2\right)$

   $\approx \$49.8$ for the 76th phone

8. Finding Speed. A body's velocity at time t sec is $s(t)=t^3-6t^2+9t-1$ m/sec. Find the body's speed each time the acceleration is zero.

$$v(t) = 3t^2 - 12t + 9 = 3(t^2-4t+3)$$
$$= 3(t-1)(t-3) \text{ m/sec}$$

$$a(t) = 6t - 12$$
$$a(t) = 0 \quad 0 = 6t - 12 \quad t = 2 \text{ sec}$$

$$\text{speed} = |v(2)| = |3(2)^2 - 12(2) + 9| = |12 - 24 + 9|$$
$$= |-3|$$
$$= 3 \text{ m/sec}$$